

Cosmic-Ray Proton to Electron Ratios

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A basic quantity in the characterization of relativistic particles is the proton-to-electron (p/e) energy density ratio. We derive a simple approximate expression suitable to estimate this quantity, $U_p/U_e = (m_p/m_e)^{(3-q)/2}$, valid when a nonthermal ‘gas’ of these particles is electrically neutral and the particles’ power-law spectral indices are equal – e.g., at injection. This relation partners the well-known p/e number density ratio at 1 GeV, $N_p/N_e = (m_p/m_e)^{(q-1)/2}$.

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1. Introduction

The proton-to-electron (p/e) number-density and energy-density ratios are very useful relations in cosmic-ray (CR) studies. Simple expressions for these ratios are commonly used, but while the standard limiting formula for the first ratio has been derived long ago (e.g., Schlickeiser 2002), there seems to be no derivation (to our best knowledge) of a similar expression for the p/e energy density ratio. Due to the basic interest in the latter ratio (e.g., in the estimation of the proton energy density from the more readily measured electron energy density), it is useful to have a convenient expression also for this ratio. This can be easily obtained when the total number of energetic protons very closely equals that of energetic electrons.

2. Number density ratio

We begin with the common assumption that suprathermal protons and electrons with initial kinetic energy $T_0 \simeq 10$ keV are accelerated to relativistic energies, attaining a power-law spectral distribution in momentum, $N_j(p) = N_{0,j} p^{-q_j}$ where $j=e,p$ for, respectively, electrons and protons; generally, $q_e \neq q_p$. If charge neutrality is preserved, the number density of each particle species is

$$n_o = \int_{T_0}^{\infty} N_e(T) dT = \int_{T_0}^{\infty} N_p(T) dT. \quad (1)$$

From $T = \sqrt{m^2 c^4 + p^2 c^2} - mc^2$, it follows that $p = \sqrt{T^2/c^2 + 2Tm}$, and $dp/dT = (T/c^2 + m)(T^2/c^2 + 2Tm)^{-1/2}$. As $N_j(T) = N_j[p(T)]dp/dT$, it follows

$$N_j(T) = \frac{N_{0,j}}{c^2} (T + m_j c^2) \left(\frac{T^2}{c^2} + 2Tm_j \right)^{-(q_j+1)/2}. \quad (2)$$

Inserting Eq.(2) in Eq.(1) we obtain the normalization of each CR species,

$$N_{0,j} = n_0 (q_j - 1) \left[\frac{T_0^2}{c^2} + 2T_0 m_j \right]^{(q_j-1)/2}. \quad (3)$$

Because of the assumed electrical neutrality of the primary CRs, we get

$$\frac{N_{0,p}}{N_{0,e}} = \frac{q_p - 1}{q_e - 1} \frac{[(T_0/c^2) + 2T_0 m_p]^{(q_p-1)/2}}{[(T_0/c^2) + 2T_0 m_e]^{(q_e-1)/2}}; \quad (4)$$

if $q_p = q_e = q$, Eq.(4) reduces to

$$N_{0,p}/N_{0,e} = (m_p/m_e)^{(q-1)/2}. \quad (5)$$

The p/e number density ratio is

$$\zeta(T) \equiv \frac{N_p(T) dT}{N_e(T) dT}. \quad (6)$$

Inserting Eqs.(2),(4) in Eq.(6), we obtain

$$\zeta(T) = \frac{(q_p - 1)}{(q_e - 1)} \frac{[T_0^2 + 2T_0 m_p c^2]^{\frac{q_p-1}{2}}}{[T_0^2 + 2T_0 m_e c^2]^{\frac{q_e-1}{2}}} \frac{T^{-(\frac{q_p+1}{2})} (T + m_p c^2) (T + 2m_p c^2)^{-\frac{q_p+1}{2}}}{T^{-(\frac{q_e+1}{2})} (T + m_e c^2) (T + 2m_e c^2)^{-\frac{q_e+1}{2}}}; \quad (7)$$

setting $q_p = q_e = q$ (e.g., at CR injection), Eq.(7) yields (Schlickeiser 2002)

$$\zeta(T) = \begin{cases} 1 & \dots T/c^2 \ll m_e \\ \propto [T/m_p c^2]^{(q-1)/2} & \dots m_e \ll T/c^2 \ll m_p \\ [m_p/m_e]^{(q-1)/2} & \dots T \gg m_p c^2 \end{cases}. \quad (8)$$

3. Energy density ratio

The p/e energy density ratio is

$$\kappa(T_0; q_p, q_e) \equiv \frac{\int_{T_0}^{\infty} N_p(T) T dT}{\int_{T_0}^{\infty} N_e(T) T dT}. \quad (9)$$

Inserting Eqs.(2),(4) in Eq.(9), we obtain

$$\begin{aligned} \kappa(T_0; q_p, q_e) &= \frac{(q_p - 1)}{(q_e - 1)} \frac{(T_0^2 + 2T_0 m_p c^2)^{\frac{q_p-1}{2}}}{(T_0^2 + 2T_0 m_e c^2)^{\frac{q_e-1}{2}}} \times \\ &\times \frac{\int_{T_0}^{\infty} T^{-\frac{q_p-1}{2}} (T + 2m_p c^2)^{-\frac{q_p+1}{2}} (T + m_p c^2) dT}{\int_{T_0}^{\infty} T^{-\frac{q_e-1}{2}} (T + 2m_e c^2)^{-\frac{q_e+1}{2}} (T + m_e c^2) dT}. \end{aligned} \quad (10)$$

In Table 1 we report values of κ for several (q_p, q_e) pairs of astronomical interest.

Denoting the integrands on the top and bottom of the r.h.s. of Eq.(10), respectively, $f_p(T)$ and $f_e(T)$, we can rewrite $\int_{T_0}^{\infty} f_p(T)dT = \int_{T_0}^{m_p c^2} f_p(T)dT + \int_{m_p c^2}^{\infty} f_p(T)dT$ and $\int_{T_0}^{\infty} f_e(T)dT = \int_{T_0}^{m_e c^2} f_e(T)dT + \int_{m_e c^2}^{\infty} f_e(T)dT$. An approximate *estimator* of κ can be obtained by considering only proton and electron energies exceeding the respective particles' rest mass. Then Eq.(10) simplifies into

$$\kappa(q_p, q_e) \simeq \frac{(q_p - 1)}{(q_e - 1)} \frac{(q_e - 2)}{(q_p - 2)} \frac{(2T_0 m_p c^2)^{\frac{q_p - 1}{2}}}{(2T_0 m_e c^2)^{\frac{q_e - 1}{2}}} \frac{(m_p c^2)^{2 - q_p}}{(m_e c^2)^{2 - q_e}}; \quad (11)$$

if $q_p = q_e = q$ (e.g., at CR injection), Eq.(11) reduces to

$$\kappa(q) \simeq \left(\frac{m_p}{m_e} \right)^{(3-q)/2}. \quad (12)$$

Table 1. Proton-to-electron energy density ratios^[a].

q_p	q_e	κ	q_p	q_e	κ	q_p	q_e	κ	q_p	q_e	κ	q_p	q_e	κ
2.0	2.0	25.8	2.1	2.0	9.84	2.2	2.0	4.18	2.3	2.0	1.97	2.4	2.0	1.01
2.0	2.1	62.8	2.1	2.1	23.9	2.2	2.1	10.2	2.3	2.1	4.79	2.4	2.1	2.46
2.0	2.2	119	2.1	2.2	45.3	2.2	2.2	19.3	2.3	2.2	9.06	2.4	2.2	4.66
2.0	2.3	189	2.1	2.3	72.0	2.2	2.3	30.6	2.3	2.3	14.4	2.4	2.3	7.40
2.0	2.4	269	2.1	2.4	102	2.2	2.4	43.6	2.3	2.4	20.5	2.4	2.4	10.5
2.0	2.5	357	2.1	2.5	136	2.2	2.5	57.8	2.3	2.5	27.2	2.4	2.5	14.0
2.0	2.6	451	2.1	2.6	172	2.2	2.6	73.1	2.3	2.6	34.4	2.4	2.6	17.7
2.0	2.7	551	2.1	2.7	210	2.2	2.7	89.2	2.3	2.7	42.0	2.4	2.7	21.6
2.0	2.8	654	2.1	2.8	249	2.2	2.8	106	2.3	2.8	49.9	2.4	2.8	25.6
2.0	2.9	760	2.1	2.9	289	2.2	2.9	123	2.3	2.9	57.9	2.4	2.9	29.8
2.0	3.0	867	2.1	3.0	330	2.2	3.0	140	2.3	3.0	66.1	2.4	3.0	34.0

^[a] Limits of integration are 10 keV–100 TeV.

4. Discussion

The assumption of electric charge neutrality when only the two main relativistic particle species are considered is essentially an approximation whose validity rests on the relatively lower abundances of other ionic species in comparison with that of protons. The energy density of relativistic electrons is readily deduced from spectral radio (synchrotron) measurements, whereas there are only few sources for which the proton energy density can be directly deduced from γ -ray measurements of the radiative decay of neutral pions produced in p-p collisions. We derived a simple expression to estimate the p/e energy density ratio as a function of q_p and q_e , suitable to estimate U_p from the observationally-deduced value of U_e – and to provide a link between protons and electrons at injection (e.g., $q_p = q_e$).

References

1. R. Schlickeiser, in *Cosmic Ray Astrophysics* (Springer-Verlag, Berlin, 2002), p. 472.